AN ADAPTIVE CONSTRAINT HANDLIG TECNIQUE FOR PARTICLE SWARM IN CONSTRAINED OPTIMIZATION PROBLEMS

UMA TÉCNICA DE TRATAMENTO DE RESTRIÇÕES ADAPTATIVA PARA ENXAME DE PARTÍCULAS EM PROBLEMAS DE OTIMIZAÇÃO COM RESTRIÇÕES

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RESUMO

Metaheurísticas inspiradas na natureza são largamente utilizadas para resolver problemas de otimização. No entanto, essas técnicas devem ser adaptadas ao resolver problemas de otimização com restrições, que são comuns em situações do mundo real. Aqui uma abordagem de penalização adaptativa (chamada Método de Penalização Adaptativa, APM) é combinada com uma técnica de Otimização por Enxame de Partículas (PSO) para resolver problemas de otimização com restrições. Esta abordagem é analisada utilizando um conjunto de problemas teste e 5 problemas de engenharia mecânica. Além disso, três variantes do APM são consideradas nos experimentos computacionais. A comparação dos resultados mostrou que o algoritmo proposto obteve um desempenho promissor na maioria dos problemas teste.
Palavras-chave: Otimização por Enxame de Partículas. Otimização com Restrições. Método de Penalização Adaptativa.

ABSTRACT

Nature inspired meta-heuristics are largely used to solve optimization problems. However, these techniques should be adapted when solving constrained optimization problems, which are common in real world situations. Here an adaptive penalty approach (called Adaptive Penalty Method, APM) is combined with a particle swarm optimization (PSO) technique to solve constrained optimization problems. This approach is analyzed using a benchmark of test-problems and 5 mechanical engineering problems. Moreover, three variants of APM are considered in the computational experiments. Comparison results show that the proposed algorithm obtains a promising performance on the majority of the test problems.
Keywords: Particle Swarm Optimization. Constrained Optimization. Adaptive Penalty Method.

1. INTRODUCTION

Optimization has been applied in many fields such as business, science, and engineering. Effective optimization techniques are important for improving the performance of applications and processes. A typical optimization problem has an objective function, equality/inequality constraints and upper/lower bounds on its decision variables. Most of the practical optimization problems are nonlinear and non-convex in either the objective and/or constraints, and so optimization of such problems requires a global optimization method (Zhang & Rangaiah, 2012).

In constrained optimization problems, one aims to minimize (or maximize) a function searching for the values of the design variables from a set of options (continuous, discrete, or mixed) which satisfy the set of constraints.

Evolutionary Algorithms (EAs) are stochastic optimization methods based on the principles of natural biological evolution (Han & Kim, 2002) and they are commonly applied to solve real-world optimization problems (Deb *et al.*, 2002). An EA that has been obtained good results in several problems in the literature is Particle Swarm Optimization (PSO) (Eberhart & Kennedy, 1995), which is a population-based algorithm for optimization based on a simplified social model that is closely tied to swarming theory. The algorithm was developed based on the social behavior of some species of birds when searching for food (Eberhart & Kennedy, 1995). The PSO approach has a simple concept and this is easily implemented. Compared with other EAs, the main advantages of PSO are its robustness in controlling parameters and its high computational efficiency (Kennedy *et al.*, 2001). A modified PSO called CRPSO and proposed by (Kar *et al.*, 2012) is adopted here in order to avoid premature convergence. The CRPSO's main feature is a new velocity expression and an operator called “craziness velocity".

Despite its robustness and global searching capacity, EAs were (originally) designed to be applied to unconstrained optimization problems. Thus, a constraint handling technique is necessary when this type of technique is applied to a constrained optimization problem.

The penalty function method has been the most popular constraint-handling technique in EAs due to its simple principle and easy implementation. The main difficulty of using a static penalty function lies in choosing appropriate values of penalty factors, which are problem-dependent (Kaveh & Talatahari, 2009). Many works in the literature discuss techniques to handle constraints with parameters chosen by the user, such as (Barbosa, 1999), (Koziel & Michalewicz, 1998), (Koziel & Michalewicz, 1999), (Orvosh & Davis, 1994) and (Runarsson & Yao, 2000). The presence of constraints significantly affects the performance of many optimization algorithms, including PSO.

APM (Adaptive Penalty Method), proposed by (Barbosa & Lemonge, 2002), is an adaptive approach to handle with constraints. APM does not require any type of user-defined penalty parameter and its penalty coefficients are calculated based on information obtained from the population, such as the average of objective function values and the level of violation of each constraint. Many works can be found in the literature where APM is used within EAs, for instance: Genetic Algorithms (Barbosa & Lemonge, 2002), Differential Evolution (Vargas *et al.*, 2013)], and PSO (Carvalho *et al.*, 2015). In addition to the original APM, several variants were proposed and analyzed by (Carvalho *et al.*, 2015) when solving constrained structural optimization problems.

The performance of APM and some of its variants are analyzed here when coupled to the CRPSO algorithm solving constrained optimization problems. Several experiments are performed and the results are analyzed and compared to those obtained by other techniques from the literature.

The paper is organized as follows. In the next section the general constrained optimization problem is described. Section 3 presents a particle swarm algorithm. A brief discussion of techniques to handle constrained optimization problems is presented in Section 4. Numerical experiments, with several test problems from the literature, are presented in Section 5. Finally, in Section 6, the conclusions and proposed future works are presented.

2. CONSTRAINED OPTIMIZATION PROBLEMS

A standard constrained optimization problem in $\mathbb{R}^n$ can be defined as

$$\min f(x) \tag{1}$$

subject to

$$g_p(x) \leq 0, p = 1, \dots, n_p \tag{2}$$

$$h_q(x) \leq 0, q = p + 1, \quad \dots, m \tag{3}$$

$$x_i^L(x) \leq x_i \leq x_i^U(x), \quad i = 1, \dots, n \tag{4}$$

where *m* is the number of constraints, *n* is the number of design variables, and $g_p(x)$ and $h_q(x)$ are the inequality and equality constraints, respectively. Usually, equality constraints are transformed into inequality ones as

$$|h_j(x)| - \epsilon \leq 0 \tag{5}$$

where ϵ is the allowed tolerance of the equality constraints.

3. PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) was proposed by (Eberhart & Kennedy, 1995). It is a population-based algorithm which has been inspired by the social behavior of animals, such as fish schooling, insects swarming and bird flocking. PSO was first applied to optimization problems with continuous variables (Parsopoulos & Vrahatis, 2002). The algorithm shows a faster convergence rate than other EAs for solving some optimization problems (Kennedy *et al.*, 2001).

In PSO, each particle of the swarm represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on the best positions of individual particles in each iteration. The objective function is evaluated for each particle and the fitness values of particles are obtained in order to determine which position in the search space is the best one.

In each iteration, the swarm is updated using the following equations (Eberhart & Kennedy, 1995)

$$v_j^{(i)}(t+1) = v_j^{(i)}(t) + c_1 \cdot r_1 \left(x_{pbest}^{(i)} - x_j^{(i)}\right) + c_2 \cdot r_2 \left(x_{gbest} - x_j^{(i)}\right) \tag{6}$$

$$x_j^{(i)}(t+1) = x_j^{(i)}(t) + v_j^{(i)}(t+1) \tag{7}$$

where $v_j^{(i)}$ and $x_j^{(i)}$ represent the current velocity and the current position of the *j*th design variable of the *i*th particle, respectively. $x_{pbest}^{(i)}$ is the best position of the *i*th particle (called *pbest*) and x_{gbest} is the best global position among all the particle in the swarm (called *gbest*); c_1 and c_2 are coefficients that control the influence of cognitive and social information, respectively, and r_1 and r_2 are two random values generated with uniform distribution between 0 and 1.

The basic PSO algorithm can be briefly described using the following steps:

1. Initialize randomly a particle swarm (positions) and velocities.
2. Initialize x_{pbest} and x_{gbest}.
3. Calculate the objective function value of each particle of the swarm.
4. Update x_{pbest} and x_{gbest}.
5. Update the position and velocity (Equations (6) and (7)).
6. Repeat the steps 3 to 5 until a stop condition is satisfied.

PSO has undergone many changes since its introduction in 1995. As researchers have learned about the technique, they have derived new versions, developed new applications, and published theoretical studies of the effects of the various parameters and aspects of the algorithm (Poli *et al.*, 2007). An improved particle swarm optimization technique called Craziness based Particle Swarm Optimization (CRPSO), proposed by (Kar *et al.*, 2012), is used here in order to get rid of the limitations of original PSO. The authors have modified the PSO by introducing an entirely new velocity expression v_i associated with many random numbers and an operator called "craziness velocity".

In CRPSO the velocity can be expressed as (Kar *et al.*, 2012)

$$v_i^{(i)} = r_2 \cdot sign(r_3) \cdot v_j^{(i)}(t) + (1 - r_2) \cdot c_1 \cdot r_1 \left(x_{pbest}^{(i)} - x_j^{(i)}\right) + (1 - r_2) \cdot c_2 \cdot (1 - r_1)\left(x_{gbest}^{(i)} - x_j^{(i)}\right) + P(r_4) \cdot sign2(r_4) \cdot v_j^{craziness} \tag{8}$$

where r_1, r_2, r_3, and r_4 are random values uniformly taken from the interval [0,1), $sign(r_3)$ is a function defined as

$$sign(r_3) = \begin{cases} -1, r_3 \leq 0.05 \\ 1, r_3 > 0.05 \end{cases} \tag{9}$$

$v_j^{craziness}$, the craziness velocity, is a user define parameter from the interval [$v^{\min}$,$v^{\max}$], $P(r_4)$and $sign2(r_4)$ are defined, respectively, as

$$P(r_4) = \begin{cases} 1, r_4 \leq Pcr \\ 0, r_4 > Pcr \end{cases} \tag{10}$$

$$sign2(r_4) = \begin{cases} -1, r_4 \geq 0.5 \\ 1, r_4 < 0.5 \end{cases} \tag{11}$$

and Pcr is a predefined probability of craziness. One can notice that while Pcr is a fixed value, $P(r_4)$ varies every time the velocity is calculated.

4. AN ADAPTIVE PENALTY TECHNIQUE

The majority of engineering design problems involves constraints. Thus, appropriate methods for constraint handling are important. Evolutionary Algorithms can be seen as unconstrained search techniques since, in their original form, they do not incorporate any explicit mechanism to handle constraints. Because of this, several authors have proposed a variety of constraint-handling techniques explicitly designed for evolutionary algorithms (Coello, 2002), (Efren, 2009) and (Kicinger *et al.*, 2005).

The most common approach in the EA community to handle constraints (particularly, inequality constraints) is to use penalty functions (Coello & Carlos, 1999). Several researchers have studied heuristics on the design of penalty functions. Probably the most well-known of these studies is the one conducted in (Richardson, 1989). The main idea is to transform a constrained optimization problem into an unconstrained one by adding a penalty function.

Penalty methods, although quite generally, require considerable domain knowledge and experimentation in each particular application in order to be effective. They can be also classified as static, dynamic and adaptive. Static penalty depends on the definition of an external factor to be added to or multiplied by the objective function. Dynamic penalty methods, in general, have penalty coefficients directly related to the number of generations, and the adaptive penalty considers the level of violation of the population by constraints during the evolutionary process. This paper does not attempt to cover the current literature on constraint handling and the reader is referred to survey papers or book chapters of e.g. (Barbosa *et al.*, 2015), (Coello, 2002), (Mezura-Montes & Coello, 2011), (Michalewicz, 1995) and (Michalewicz & Schoenauer, 1996).

An adaptive penalty method (APM) was originally introduced by (Barbosa & Lemonge, 2002). The method uses information from the population, such as the average of the objective function and the level of violation of each constraint during the evolution. When using APM, the fitness function can be written as

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible} \\ \bar{f}(x) + \sum_{j=1}^{m} k_j v_j(x), & \text{otherwise} \end{cases} \tag{12}$$

where

$$\bar{f}(x) = \begin{cases} f(x), & \text{if } f(x) > \langle f(x) \rangle \\ \langle f(x) \rangle, & \text{if } f(x) \le \langle f(x) \rangle \end{cases} \tag{13}$$

and $\langle f(x) \rangle$ is the average of the objective function values in the current population.

The penalty parameter k_j is defined at each generation as

$$k_j = \frac{|\langle f(x) \rangle| \langle \delta_j(x) \rangle}{\sum_{l=1}^{m} [\langle \delta_l(x) \rangle]^2} \tag{14}$$

where $\langle \delta_j(x) \rangle$ is the violation of the *j*th constraint averaged over the current population.

Three variants of APM are analyzed here:

- Variant 3 (Barbosa & Lemonge, 2008): no penalty coefficient k_j is allowed to have its value reduced along the evolutionary process, if $k_j^{new} < k_j^{current}$ then $k_j^{new} = k_j^{current}$.

- Variant 5 (Carvalho *et al.*, 2015): $\overline{f(x)}$ is modified as

$$\bar{f}(x) = \begin{cases} f(x), & if\ f(x) > \lfloor f(x) \rfloor \\ \lfloor f(x) \rfloor, & otherwise, \end{cases} \tag{15}$$

where $\lfloor f(x) \rfloor$ is the value of the objective function of the worst feasible individual (the average of the objective function values is used when no feasible individual exist).

- Variant 7 (Carvalho *et al.*, 2015): $\langle v_j(x) \rangle$, which originally represented the average of the violations of all individuals at each constraint, is defined here as the sum of the violation of all individuals which violate the *j*-th constraint divided by the number of individuals which violate this constraint. Also, $\langle f(x) \rangle$, which represented the average of the objective function values, now denoted by $\langle\langle f(x) \rangle\rangle$, is the sum of the objective function values of all individuals in the current population divided by the number the infeasible individuals. Thus, k_j is defined in Variant 7 as

$$k_j = |\langle\langle f(x) \rangle\rangle| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{m} [\langle v_l(x) \rangle]^2}. \tag{16}$$

5. NUMERICAL EXPERIMENTS

In this section, the performance of the CRPSO when using APM or one of its variants is investigated on well-known and widely used test problems. The test-functions used in the computational experiments includes a set of 24 functions known as G-Suite (Liang *et al.*, 2006) and 5 mechanical engineering optimization problems (Bernardino *et al.*, 2007). Only feasible solutions were found in the 35 independents runs.

The parameters which were used for the CRPSO algorithm was a swarm size equal to 50, $v_i^{craziness} = 0.001$, $c_1 = c_2 = 2.05$, and $Pcr = 0.5$. For Variant 3, the penalty parameter *k* is updated every 10 generations. It should be understood that discrete or integer design variables are considered as the nearest integer of the corresponding variable of the vector solution (particle). It points to either an index of a table of discrete values or an integer.

5.1. Performance analysis of experiments

The experiments are compared using the performance profiles proposed by (Dolan & Moré, 2002), an analytical resource for the visualization and interpretation of the results obtained in the numerical experiments.

Given a set P of test problems p_j, with $j = 1,2,\ldots,n_p$, a set of algorithm a_i with $i = 1,2,\ldots,n_a$, and $t_{p,a} > 0$ a performance metric (for instance, computational time), the performance ratio is defined as

$$r_{p,a} = \frac{t_{p,a}}{\min\{t_{p,a}: a \in A\}}. \tag{17}$$

Thus, the performance profile of the algorithm *a* is defined as

$$\rho_a(\tau) = \frac{1}{n_p} \left| \{ p \in P : r_{p,a} \leq \tau \} \right| \qquad (18)$$

where $\rho_a(\tau)$ is the probability that the performance ratio $r_{p,a}$ of algorithm $a \in A$ is within a factor $\tau \geq 1$ of the best possible ratio. If the set *P* is a good representation of problems to be addressed, then algorithms with larger $\rho_a(\tau)$ are to be preferred. The performance profiles have a number of useful properties (Barbosa *et al.*, 2010) and (Dolan & Moré, 2002). Other studies using performance profiles in performance analysis of algorithms can be found in (Barbosa *et al.*, 2010) and (Bernardino *et al.*, 2011).

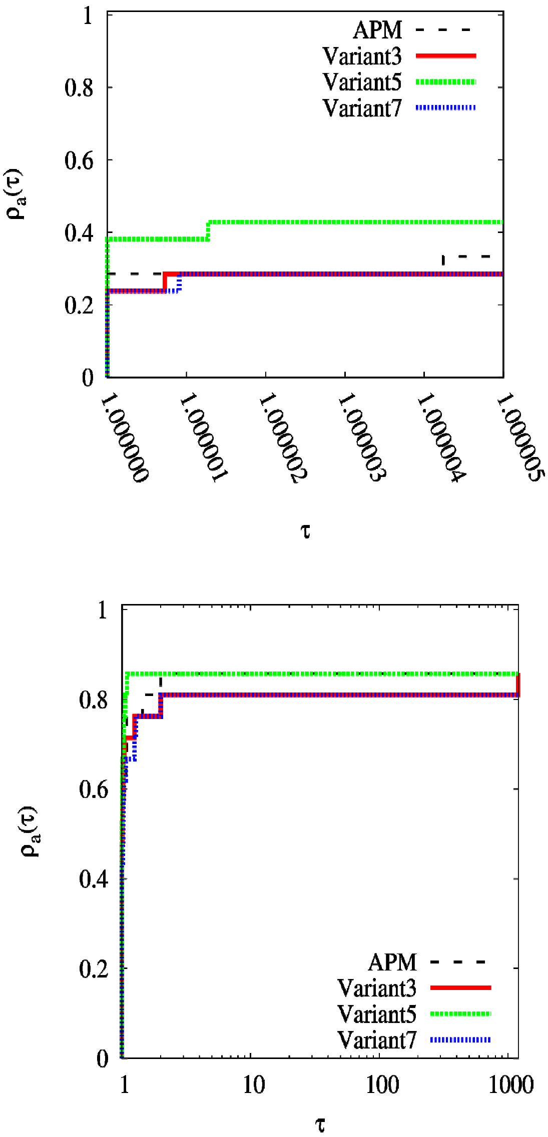
5.2. G-Suite

The first experiment is based in a popular suite of function presented in (Liang *et al.*, 2006). The experiments were performed considering three levels of evaluations of the objective function: 5000, 50000 and 500000, commonly used in the literature for these experiments. The summary of the 24 test problems is given in Table 1 where *n* is the number of design variables, ρ is the estimated rate between the feasible region and the search space, and *ni* and *ne* are the number of inequality and equality constraints, respectively.

Table 1: Summary of the 24 functions of G-Suite.

Problem	*N*	Type of $f(x)$	ρ (%)	*ni*	*ne*
G01	13	quadratic	0.0111	9	0
G02	20	non-linear	99.9971	2	0
G03	10	polynomial	0.0000	0	1
G04	5	quadratic	52.1230	6	0
G05	4	cubic	0.0000	2	3
G06	2	cubic	0.0066	2	0
G07	10	quadratic	0.0003	8	0
G08	2	non-linear	0.8560	2	0
G09	7	polynomial	0.5121	4	0
G10	8	linear	0.0010	6	0
G11	2	quadratic	0.0000	0	1
G12	3	quadratic	4.7713	1	0
G13	5	non-linear	0.0000	0	3
G14	10	non-linear	0.0000	0	3
G15	3	quadratic	0.0000	0	2
G16	5	non-linear	0.0204	38	0
G17	6	non-linear	0.0000	0	4
G18	9	quadratic	0.0000	12	0
G19	15	non-linear	33.4761	5	0
G20	24	linear	0.0000	6	14
G21	7	linear	0.0000	1	5
G22	22	linear	0.0000	1	19
G23	9	linear	0.0000	2	4
G24	2	linear	79.6556	2	0

The average of the objective function values is used here as performance metric for the three levels of budget. Figure 1(a) shows the performance profiles in the range $\tau \in [1; 1.000005]$ when 5000 objective function evaluations are allowed. In this case, Variant 5 presented the highest value of $\rho(1)$, indicating that this variant obtained the best performance in a larger number of problems. In Figure 1(b) it is possible to see that Variant 5 obtained the lowest value of τ such as $\rho(\tau)$ assumes the largest value in that plot; this suggests that Variant 5 is also the most robust method between those analyzed here.

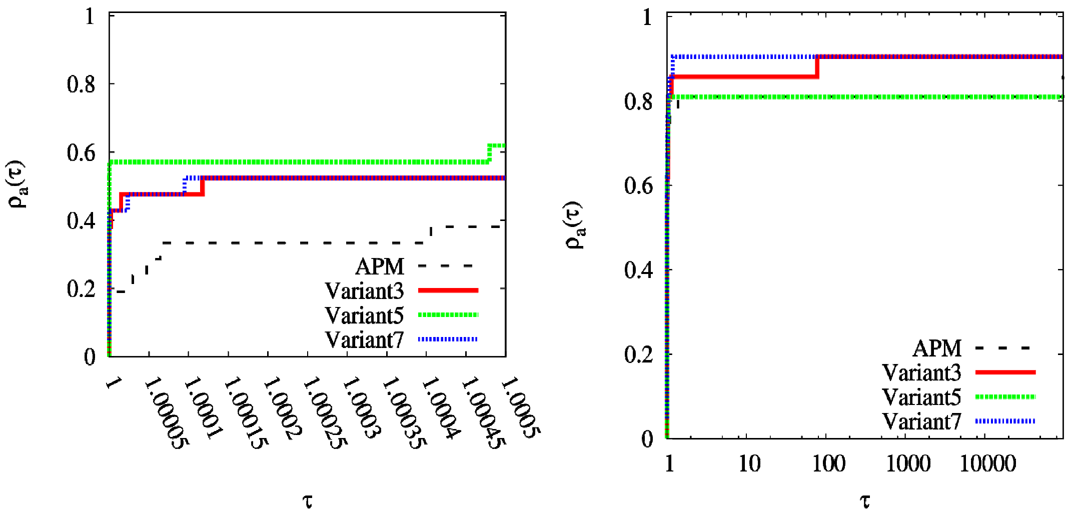


(a)

(b)

Figure 1: Performance profiles of the results obtained when solving G-Suite – 5000 objective function evaluations.

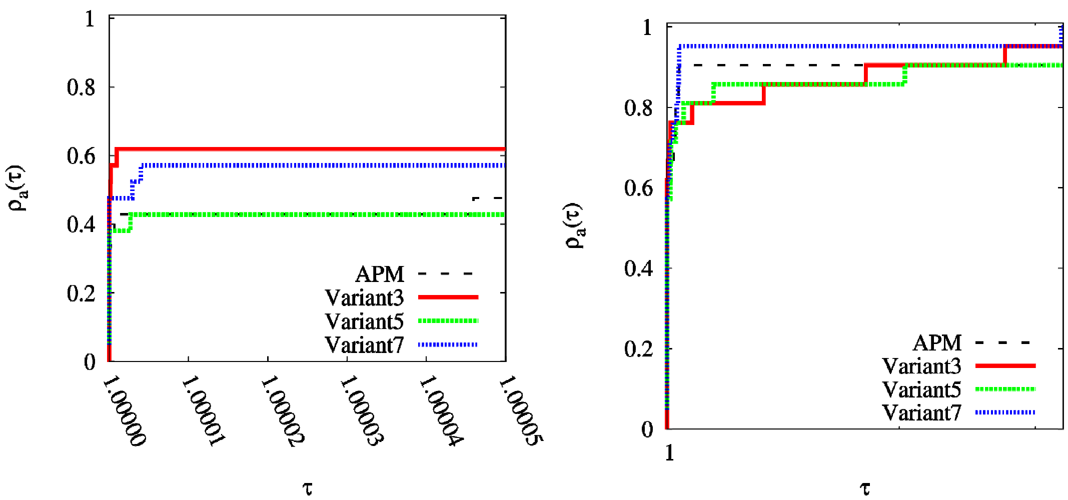
The performance profiles of the results obtained when using the 50000 objective function evaluations can be found in Figure 2(a) in which one can note that Variant 5 presented the best performance in the majority of the 24 test-problems. In Figure 2(b), one can seen that Variant 7 obtained the most robust solutions.



(a) (b)

Figure 2: Performance profiles of the results obtained when solving G-Suite – 50000 objective function evaluations.

Finally, according to the performance profiles shown in Figure 3(a), Variant 3 presented the best performance in the majority of problems. On the other hand, Variant 7 presented more robustness, as one can see in Figure 3(b).



(a) (b)

Figure 3: Performance profiles of the results obtained when solving G-Suite – 500000 objective function evaluations.

Besides the identification of the best performing technique in the majority of problems and the robustness one, performance profiles can be also used to indicate the method with the best performance in general; this can be made by means of the area under of the performance profiles curves (higher values are preferable). Table 2 present the normalized areas under the performance profiles curves for G-Suite, where *nfe* means the number of function evaluations. It can be observed that the best variant (in general) for this set of 24 test-problems is Variant 7 which achieved 1 when using the 50000 and 500000 objective function evaluations, followed by the Variant 5 which achieved 1 when using 5000.

Table 2: Normalized areas under the performance profiles curves for G-Suite with 5000, 50000 and 500000 objective function evaluations.

	Area		
	5000 *nfe*	50000 *nfe*	500000 *nfe*
APM	0.99992	0.89736	0.95185
Variant 3	0.94439	0.99995	0.90824
Variant 5	1	0.894736	0.92695
Variant 7	0.94437	1	1

5.3. Mechanical engineering problems

Five mechanical engineering problems are also used in this paper to evaluate the performance of the algorithm when using APM or one of its variants. Table 3 presents some details of each test problem where *n* is the number of design variables, *ni* and *ne* are the number of inequality and equality constraints, respectively and *nfe* is the number of function evaluations.

Table 3: Mechanical engineering problems.

Problem	*n*	Type of $f(x)$	*ni*	*ne*	*nfe*
Welded Beam (WB)	4	quadratic	5	0	32000
Pressure Vessel (PV)	4	cubic	4	0	80000
Cantilever Beam (CB)	10	quadratic	11	0	35000
Speed Reducer (SR)	7	quadratic	11	0	36000
T./C. Spring (TCS)	3	non-linear	4	0	36000

In reference (Bernardino *et al.*, 2007) it is possible obtain the description of the mechanical engineering problems used here. Figs. 4 to 8 illustrate each test-problem.

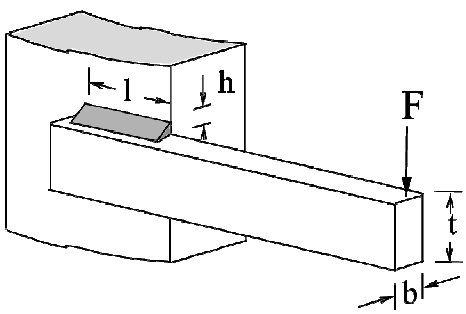


Figure 4: The Welded Beam.

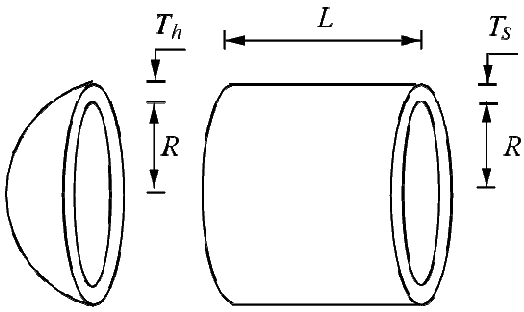


Figure 5: The Pressure Vessel.

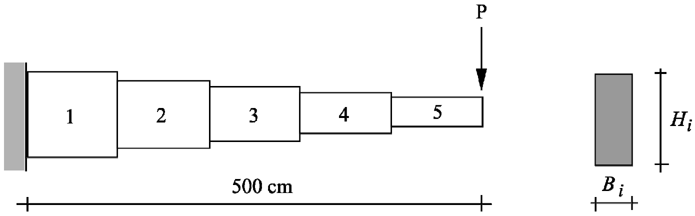


Figure 6: The Cantilever Beam.

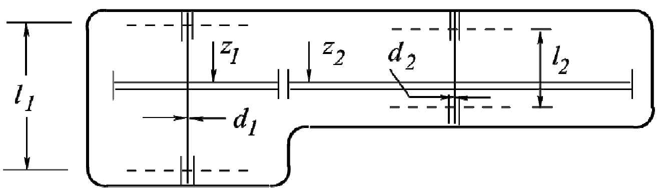


Figure 7: The Speed Reducer.

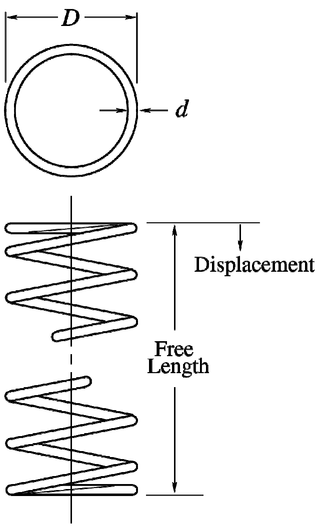


Figure 8: The Tension/Compression Spring.

The results obtained for the mechanical engineering problems are presented in Tables 4, 6, 8, 10 and 12, where the best ones are displayed in **boldface**, and *std* means the standard deviation and *nesf* means the total number of independent runs in which feasible solutions were found. A hybridization of a Genetic Algorithm with an Artificial Immune System is proposed in (Bernardino *et al.*, 2008) and its results are used here in the comparisons. It can be observed that in four of the five test-problems the APM or one of its variants achieved the best solution. Tables 5, 7, 9, 11 and 13 presents the design variables considering the best result for each problem for “This study” and reference (Bernardino *et al.*, 2008).

Table 4: Values found for Tension/Compression Spring design.

Method	Best	Median	Average	Std	Worst	nesf
APM	0.01267	0.01311	0.01354	6.9509e-03	0.01742	35/35
Variant 3	**0.01266**	0.01306	0.01392	8.6313e-03	0.01734	35/35
Variant 5	0.01267	**0.01288**	0.01393	9.6090e-03	0.01777	35/35
Variant 7	**0.01266**	0.01312	0.01389	9.1731e-03	0.01777	35/35
(Bernardino *et al.*, 2008)	**0.01266**	0.01289	**0.01313**	**6.2800e-04**	**0.01531**	50/50

Table 5: Comparison of results for Tension/Compression Spring design.

	d	D	N	Volume
This study	0.05406	0.41655	8.48436	**0.01266**
(Bernardino *et al.*, 2008)	0.05143	0.35053	11.6612	0.01267

Table 6: Values found for Speed Reducer design.

Method	Best	Median	Average	std	Worst	nesf
APM	2996.3592	2996.3837	2998.8105	2.6529e+01	3007.4698	35/35
Variant 3	2996.3631	3005.7006	3003.6417	4.4687e+01	3016.7882	35/35
Variant 5	2996.3654	2996.3927	3002.1440	4.2637e+01	3016.7988	35/35
Variant 7	2996.3622	2996.3780	2999.6083	3.4911e+01	3016.7808	35/35
(Bernardino *et al.*, 2008)	**2996.3483**	**2996.3495**	**2996.3501**	**7.4500e-03**	**2996.3599**	50/50

Table 7: Comparison of results for Speed Reducer design.

	b	m	n	l_1	l_2	d_1	d_2	Weight
This study	3.5000	0.7000	17	7.3009	8.2999	3.3502	5.2868	2996.3592
(Bernardino *et al.*, 2008)	3.5000	0.7000	17	7.3000	7.8000	3.3502	5.2868	**2996.3483**

Table 8: Values found for Welded Beam design.

Method	Best	Median	Average	Std	Worst	nesf
APM	**2.38113**	2.77504	2.81474	2.2005e+00	3.69870	35/35
Variant 3	2.38118	2.51280	3.14079	1.3686e+01	16.30031	35/35
Variant 5	2.38115	2.66995	2.70019	1.8796e+00	**3.32986**	35/35
Variant 7	2.38114	**2.43315**	**2.67102**	2.0656e+00	3.46638	35/35
(Bernardino *et al.*, 2008)	2.38335	2.92121	2.99298	**2.0200e-01**	4.05600	50/50

Table 9: Comparison of results for Welded Beam design.

	h	l	t	b	Cost
This study	0.2444	6.2183	8.2912	0.2444	**2.3811**
(Bernardino *et al.*, 2008)	0.2443	6.2186	8.2914	0.2443	2.3833

Table 10: Values found for Pressure Vessel design.

Method	Best	Median	Average	std	Worst	nesf
APM	**6059.7143**	**6090.5263**	6474.8760	3.1086e+03	7544.4925	35/35
Variant 3	**6059.7143**	**6090.5263**	**6352.0563**	2.5773e+03	7544.4925	35/35
Variant 5	**6059.7143**	6318.9481	6359.9781	2.2021e+03	7544.4925	35/35
Variant 7	**6059.7143**	6370.7797	6427.6676	2.6221e+03	7544.4925	35/35
(Bernardino *et al.*, 2008)	6059.8546	6426.7100	6545.1260	**1.2400e+02**	**7388.1600**	50/50

Table 11: Comparison of results for Pressure Vessel design.

	T_s	T_h	R	L	Weight
This study	0.8750	0.4375	45.3367	140.2538	**6059.7143**
(Bernardino *et al.*, 2008)	0.8125	0.4375	42.0973	176.6509	6059.8546

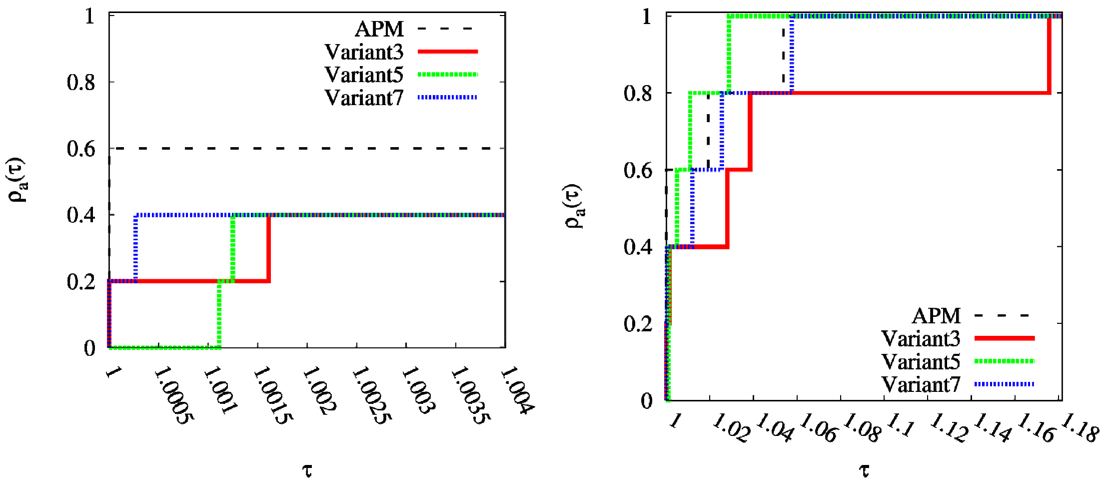
Table 12: Values found for Cantilever beam design.

Method	Best	Median	Average	Std	Worst	nesf
APM	64965.071	67943.462	**67901.329**	1.6162e+04	75143.537	35/35
Variant 3	64584.132	68673.296	70516.546	4.2187e+04	106637.832	35/35
Variant 5	**64578.229**	**67943.452**	68240.218	1.6177e+04	**73943.453**	35/35
Variant 7	64578.271	68294.702	71817.816	1.0431e+05	173520.325	35/35
(Bernardino *et al.*, 2008)	64834.700	74987.160	76004.240	**6.9300e+03**	102981.060	50/50

Table 13: Comparison of results for Cantilever beam design.

	b1	*h1*	*b2*	*h2*	*b3*	*h3*	*b4*	*h4*	*b5*	*h5*	Volume
This study	4	60	3.1	55	2.6	50	2.204	44.091	1.749	34.995	**64578.229**
(Bernardino *et al.*, 2008)	3	60	3.1	55	2.6	50	2.294	42.215	1.825	35.119	64834.700

The average of the results obtained by the variants is also used as performance metric in mechanical engineering problems. It can be observed in Figure 9(a), in the range $\tau \in [1; 1.004]$, that APM is the variant with better performance in the majority of problems. Also, one can see that although Variant 5 obtained the best average value in none of the engineering optimization problems considered here (Figure 9(a)), this achieved the most robust results (Figure 9(b)).



(a) (b)

Figure 9: Performance profiles for mechanical engineering problems.

Table 9 present the normalized areas under the performance profiles curves for mechanical engineering problem. Variant 5 presented the high value for the normalized area under the performance profiles curves, followed by APM.

Table 9: Normalized areas under the performance profiles curves for mechanical engineering problems.

Method	Area
APM	0.96866
Variant 3	0.76332
Variant 5	1
Variant 7	0.94193

5.4. Summary of the results

One can observe that the standard version of APM and its variants performed well when applied to both, G-Suite and mechanical engineering problems. Variant 7 is the best performing technique (in general) when solving the G-Suite test-function (Table 2). Also, it is important to highlight that Variant 5 presented better results (in general) when this is compared to other APM variants and solving mechanical engineering problems (Table 9). Finally, one can notice that Variant 5 achieved the largest area under the performance profiles curves when only 5000 objective function evaluations are allowed.

6. CONCLUSION

A particle swarm optimization algorithm coupled with a method to handle with constraints called APM and three of its variants are tested in a well known set of constrained optimization problems in mathematical and mechanical engineering. A comparison with an alternative approach from the literature was performed and the PSO presented here provided competitive results in the computational experiments.

The results of the computational experiments for G-Suite demonstrate that Variant 7 and Variant 5 are more robust than Variants 3 and APM. Considering an analysis using the performance profiles, Variant 7 performed better than to the others variants for G-Suite. For the mechanical engineering problems, Variant 5 showed better results compared to the other variants.

For future works the proposed method is going to be applied to real world problems from engineering considering more complexity with respect to objective functions and constraints.

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