Regular Polyhedra-Shaped Structures: mathematical concepts and applications in the Sciences

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Abstract

The interdisciplinary teaching represents an important tool for meaningful learning in Mathematics, with a view to contextualizing and applying concepts in other areas of knowledge. In this context, official educational regulations in Brazil in the area of Mathematics and its Technologies defend the construction of a more integrated vision of Mathematics and its relationship with the Science. Among the skills highlighted for the teaching of Mathematics in Basic Education, the interpretation of phenomena and the solution of problem situations based on concepts of regular polyhedra stand out. Although the importance of this topic for improving the teaching and learning of Mathematics is known, the applications of regular polyhedra are still little explored in the school environment and even in literature. In this sense, this work presents a literature review, of an exploratory-descriptive nature, with the aim of highlighting regular polyhedra in the modeling of phenomena in scientific and technological areas for use of this relationship in the educational environment. The results showed that the geometric elements and properties of regular polyhedra are used to model phenomena in areas of basic science (biology, chemistry and materials) and applied science (robotics, cartography, meteorology and mineralogy), meaning an alternative approach to the improvement of the Mathematics and Technologies teaching in secondary school.

Keywords: teaching Mathematics; spatial geometry; regular polyhedra; Science and technology; interdisciplinarity.

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Introduction

The low learning rates of Mathematics in Brazilian Basic Education are often related to the preferential use of traditional teaching methodologies (which focus only on teaching formal definitions and demonstrations), to the detriment of the use of alternative methodologies that can provide significant knowledge for solving real contemporary problems (Moreira; Silva; Alves, 2021; Santos; Oliveira; Cardoso, 2023).

However, the National Education Guidelines and Bases Law (Brasil, 1996)) was very direct on this issue, when it defined the following objectives to be achieved in secondary education: the improvement of the student as a human being; his/her ethical training, the development of their intellectual autonomy and critical thinking; and their preparation for work and the development of skills to continue their learning. In this discussion, it is important to consider the different purposes of mathematical training in basic education, including the modeling of phenomena in other areas of knowledge, and its importance for scientific development (Barbosa, 2024).

In this sense, the interdisciplinarity teaching can mean an alternative methodology in quest of meaningful learning in Mathematics. According to Do Nascimento, Pereira, and Shaw (2020, p. 144), "interdisciplinary studies involve the integration of disciplines, whose knowledge is shared/integrated, in order to solve a problem or project". In addition, the National Common Curricular Base (BNCC) defines the set of essential learning that all students must develop throughout the stages and modalities of Basic Education, organized by areas of knowledge. According to BNCC in the area of Mathematics and its Technologies, "students must build a more integrated vision of Mathematics, Mathematics with other areas of knowledge, and the application of Mathematics to reality" (Brasil, 2018, p. 471).

Among the abilities highlighted as a result of significant Mathematics learning in Basic Education, it is possible to highlight the knowledge and application of spatial geometry concepts in understanding and solving problem situations. Spatial geometry can be understood as the branch of Mathematics that studies three-dimensional figures in space. In this context, regular polyhedra (tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron) have received a lot of attention due to the numerous relationships between their three-dimensional shapes and elements from other areas of knowledge. The study of regular polyhedra is justified by the possibility of mathematical modeling and explanation of various routine situations, natural phenomena and the understanding of the relationships between these geometries and other areas of basic and applied science (Bhatt; Kumar, 2017; Ghasemi et al., 2020; Mckenna, 2009; Parvez, 2020; Peixoto, 2013; Liu et al., 2022).

Although the topic has been widely discussed in the literature, its application in schools still does not occur with the expected scope. In view of the above, an exploratory-descriptive literature review was developed, with the main objective of answering the following research question: which phenomena and problem situations are described in scientific and technological areas (and which areas are reported) based on mathematical modeling with regular polyhedra, and that have potential to be explored in the educational environment?

The results showed that the geometries and properties of regular polyhedra are closely related to the understanding of various phenomena, both in areas of basic science, such as biology, chemistry and materials, and in areas of applied science, such as robotics, cartography, meteorology, mineralogy, and nanotechnology. The literature selected for the research returned that aspects such as surface area, volume, and specific surface area of these regular geometric shapes are widely used for the description and interpretation of the applications highlighted in these studies.

Mathematical concepts about regular polyhedra

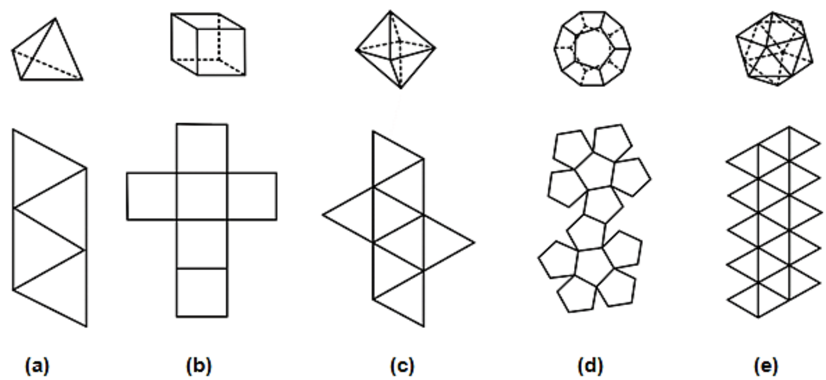
A polyhedron can be defined as a closed and limited set of space, with a non-empty interior and whose border consists of the union of a finite number of polygons. A more complete definition can be given as:

> Consider a set G obtained by the assembly of n polygons, with n ≥4, such that: any two of these polygons that have a side in common are not coplanar; each side of any of these polygons is a side of two and only two of them. The set G is called a closed polyhedral surface. This surface separates space into two regions, one of which is limited. The union of the surface G with this limited region in space is called polyhedron (Paiva, 2015, p. 188).

The three main elements of polyhedra are: face, edge and vertex. Face is the name given to each of the polygonal regions that make up the polyhedral surface. Edge is the common side of adjacent polygons that delimit two faces of the polyhedron. The vertex is the point common to three or more edges that also corresponds to the vertices of the face polygons. The nomenclature of polyhedra is given according to the number of their faces or vertices.

Polyhedra are called regular when their faces are regular polygons of the same type. Demonstrations of the proofs of these statements appeared around 300 B.C., in Book XIII of Euclid's Elements. Propositions 13 to 17 of Book XIII of Euclid's Elements describe the constructions of regular polyhedra, while the proposition 18 demonstrates that there are only five regular polyhedra, namely: tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron (Figure A, above). Their respective polyhedral surfaces in the plane are also illustrated in Figure 1. Frame 1 presents the nomenclature of regular polyhedra associated with their main geometric elements.

Figure 1: From (a) to (e), above, the regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron, and their respective polyhedral surfaces in the plane.



Source: Developed by the authors (2025).

Archimedes of Syracuse is credited with discovering semi-regular polyhedra:

> It is certain that not all of Archimedes' works have come down to us, since from a later period we know (from Pappus) that Archimedes discovered all thirteen possible so-called semi-regular solids. While a regular polyhedron has faces that are regular polygons of the same type, a semi-regular solid is a convex polyhedron whose faces are regular polygons but not all of the same type (BOYER, 1974, p. 89).

Frame 1: Nomenclatures of regular polyhedra associated with their geometric elements.

Number of edges on each face	Number of edges concurrent in a vertex	Number of vértices	Number of edges	Number of faces	Nomenclature
3	3	4	6	4	Tetrahedron
4	3	8	12	6	Hexahedron
3	4	6	12	8	Octahedron
5	3	20	30	12	Dodecahedron
3	5	12	30	20	Icosahedron

Source: Developed by the authors (2025).

There are thirteen semi-regular polyhedra, also known as Archimedean polyhedra, which are obtained through transformations on the five regular polyhedra.

Through the truncation operation, which consists of dividing the edges of the polyhedron into equal parts and constructing new vertices at these points, the following semi-regular polyhedra are obtained: truncated tetrahedron, cuboctahedron, truncated cube, truncated octahedron, rhombicuboctahedron, truncated cuboctahedron, icosidodecahedron, truncated dodecahedron, truncated icosahedron, rhombicosidodecahedron, and truncated icosidodecahedron (Figure 2).

Through the snubfication operation, which according to Soares (2021, p. 60) is an "operation that consists of moving the faces of a regular polyhedron, rotating them or not, and filling the empty spaces with regular polygons", the last two semi-regular polyhedra are obtained: cube snub and dodecahedron snub (Figure 2).

Other important concepts in the study of regular polyhedra and their applications are surface area (A), volume (V) and specific surface area. These studies involve knowledge of the planning of the regular polyhedra, as shown in Figure A. Considering that the edge of each polyhedron has length l, the area of the polyhedral surface can be determined by the product of the number of faces by the area of each of the faces of the regular polyhedron. Thus:

$A_{\text{tetrahedron}} = 4\left(l^2 \frac{\sqrt{3}}{4}\right) = l^2\sqrt{3}$; (Equation 1)

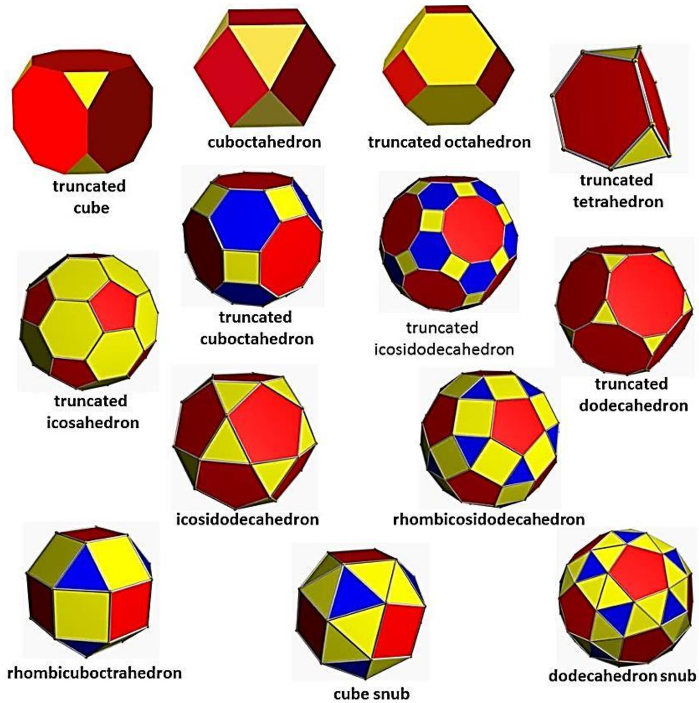
$A_{\text{hexahedron}} = 6l^2$; (Equation 2)

$A_{\text{octahedron}} = 8\left(l^2 \frac{\sqrt{3}}{4}\right) = 2l^2\sqrt{3}$; (Equation 3)

$A_{\text{dodecahedron}} = 3l^2\sqrt{25 + 10\sqrt{5}}$; (Equation 4)

$A_{\text{icosahedron}} = 20\left(l^2 \frac{\sqrt{3}}{4}\right) = 5l^2\sqrt{3}$. (Equation 5)

Figure 2: From (a) to (e), above, the regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron, and their respective polyhedral surfaces in the plane.



Source: Webb (2024).

In terms of volume, it is observing that the regular tetrahedron is a pyramid with a triangular base. Let A_b be the area of the base and h the height of the regular tetrahedron, its volume is given by $V_{tetrahedron}= \frac{A_b.h}{3}$. As $A_b= l^2\frac{\sqrt{3}}{4}$ and $h_{tetrahedron}= \frac{l\sqrt{6}}{3}$, the volume can be given by:

$$V_{\text{tetrahedron}} = \frac{l^3}{12}\sqrt{2} \qquad \text{(Equation 6)}$$

As the regular hexahedron is a prism with a square base of side l and height h=l,

$$V_{\text{hexahedron}} = l^3 \qquad \text{(Equation 7)}$$

The regular octahedron can be divided into 2 pyramids with a square base. As $A_b= l^2$ and h is equal to the radius of the circumscribed sphere, the volume of the regular octahedron is

$$V_{\text{octahedron}} = \frac{l^3\sqrt{2}}{3}. \qquad \text{(Equation 8)}$$

Similarly, the regular dodecahedron can be divided into 12 pyramids with a pentagonal base. Therefore, its volume can be given by

$$V_{\text{dodecahedron}} = 12\frac{A_b.h}{3} = \frac{l^3\,(15+7\sqrt{5})}{4}. \qquad \text{(Equation 9)}$$

In turn, the regular icosahedron can be divided into 20 pyramids with a triangular base, and its volume is

$$V_{\text{icosahedron}} = \frac{5l^3(3+\sqrt{5})}{12}. \qquad \text{(Equation 10)}$$

The relationship between surface area and volume is known as specific surface area. Considering the same edge length for each of the regular polyhedra, it is observed that the tetrahedron has the smallest surface area, smallest volume and largest specific surface area and that the dodecahedron has the largest surface area, largest volume, and smallest specific surface area. In short, the shape and size of an object or particle define its surface area and volume, directly influencing its specific surface area. Therefore, the study of the specific surface area on a nanometer scale has great relevance for researchers in various applications such as catalysts, photocatalysts, sensors, biosensors, solar cells, etc., since in these cases a greater surface area/volume ratio contributes to the optimization of properties and applications (Cheriyamundath; Vavilala, 2021).

Methodology

This work presents an exploratory-descriptive literature review, carried out through the stages of investigation and explanatory analysis of solutions and integrative synthesis (Lima; Mioto, 2007).

In the solutions investigation stage, the Google Scholar database was used to search for academic/scientific works such as these, dissertations, books, book chapters and scientific articles related to the topic under study. As a semantic search criterion, the keywords “mathematics teaching”, “regular polyhedron”, “applications”, and “modeling” were used in combination (in English or Portuguese) to start the bibliographical search. In the explanatory analysis stage of the solutions, the contributions of the various authors and works from the initial research were organized, and then they were selected based on the significance of the study for the discussion and interpretation of the results.

The inclusion criteria for defining the sample of scientific works related to the research topic considered the presence of mathematical concepts about regular polyhedra to explain phenomena in areas of knowledge, which can be addressed in interdisciplinary studies in secondary school (Brasil, 2018). The selected works are described in Frame 2.

Frame 2: Selected sample of scientific works for this research, and related applications by area.

Citation	Scientific/ technological area	Main related applications
(Jin; Gu, 2022); (Alvarez, 2021); (Coelho, 2020); (Gisbert-Gonzalez *et al*., 2018); (Bhatt; Kumar, 2017); (Mckenna, 2009); (Gracia-Pinilla *et al*., 2008)	Chemistry, Materials Science, and Nanotechnology	Molecular geometry and crystalline structure; allotropic forms of carbon; energetic and thermal properties of nanomaterials; sustainable chemistry
(Manzanares, 2023); (Ghasemi *et al*., 2020); (Ghasemi *et al*., 2019); (Pereira, 2010)	Mineralogy/ Concrete technology	Crystalline structure; shape of minerals; physical properties and mechanical behavior of soils; workability and water demand of building materials
(Lu; Guo; Liu, 2024); (Lee *et al*., 2022); (Van Beveren, 2022); (Mishra, 2020); (Parvez, 2020); (Samantha *et al*., 2020); (Rivero, 2019); (Haeckel, 2012)	Biology	Shape of living organisms; genetic code (DNA/RNA) interpretation; morphology of viruses and development of antiviral drugs
(Liu; Yao; Li, 2020); (Motahari-Bidgoli *et al*., 2014); (Ding; Yao, 2013)	Robotics	Motion and rolling mechanism technologies for robots

(Stasi, 2022); (Silva *et al.*, 2009)	Meteorology/ Cartography	Cartographic projections and maps; meteorological models

Source: Developed by the authors (2025).

In the integrative synthesis stage, the main geometry elements and properties of regular polyhedra, and its scientific applications were described and discussed, using a language suitable for the target audience (Lima; Mioto, 2007).

Applications of polyhedral modeling in basic and applied science

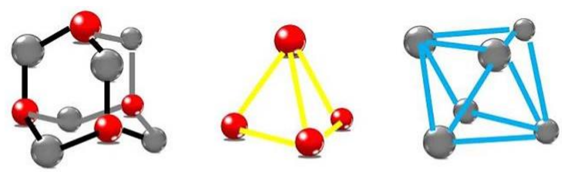
In this section, the results of the exploratory-descriptive research described in the methodological section are presented.

Chemistry, Materials Science, and Nanotechnology

Chemistry can be understood as the science that studies the composition, structure, properties of matter and the changes it undergoes. In addition, stereochemistry is one of the branches of Chemistry that studies the three-dimensional aspects of molecules. In this way, the study of molecular geometry relates Chemistry to Mathematics. In other words, the physical and chemical properties of substances can be strongly dependent on the way in which the atoms of the elements (that constitute them) are arranged in space and the type of bond that holds these atoms together.

Alvarez (2021) stated that the tetrahedron and octahedron are the most common structural models in transition metal chemistry and are considered as the basic structural units of most inorganic solids. However, other sets of polyhedral, obtained from the truncation of regular forms, can be established to represent molecular arrangements. In this sense, some regular polyhedra truncation operations are used to construct semi-regular polyhedra that can be used to represent chemical structures. By convention, the atoms of these molecules are represented by the vertices and the chemical bonds by the edges of these polyhedra. From the same perspective, it is also possible to observe the decomposition of a molecular arrangement into secondary structures that have the symmetry of regular polyhedra (Alvarez, 2021). For example, the tertiary and secondary carbon atoms in the chemical structure of adamantane ($C_{10}H_{16}$, an organic compound) are organized according to tetrahedron and octahedron symmetries, respectively (Figure 3). This behavior is also observed in binary intermetallic compounds, such as $Zn_{12}Mg_4$ and $Cu_{12}Mg_4$, which can be decomposed into regular and semiregular polyhedra.

Figure 3: From (a) to (e), above, the regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron, and their respective polyhedral surfaces in the plane.



Source: Developed by the author (2024).

The geometry of the C_{60} fullerene molecule (an allotropic type of carbon) is formed by 60 carbon atoms, where each of these atoms is linked to three other carbons. This polyhedron has 12 regular pentagonal faces, 20 regular hexagonal faces, 60 vertices, 90 edges, and has the shape of a truncated icosahedron. The smallest fullerene molecule, C_{20}, has the shape of a regular dodecahedron, a polyhedron with 20 vertices, 30 edges, 12 pentagonal faces, and no hexagonal faces (Jin; Gu, 2022).

Fullerenes stand out as the third most stable allotropic form of carbon, used in the synthesis of new chemical compounds with improved functionality. As shown, the study of the geometric shapes of fullerenes highlights the mathematical-chemical relationship of their structures and allows us to discuss the interpretation of their properties depending on their molecular arrangements. These are some of the studies in which Chemistry is essential for the development of the area of Materials Science. Materials Science is the area of knowledge that addresses the relationships between the structure of a material and its properties. In this context, it is important to highlight that the properties of the matter depend not only on its composition and structure, but also on its dimensional scale. The same material at different scales can present different properties. For example, the shape and size of a particle directly influence its specific surface area and, consequently, the interaction of the material with other materials and/or the environment.

Coelho (2020, p.7) also highlights that in the nanometric scale (of the order of 10^{-9} m) "materials tend to present very special properties when interacting with a light source and, as they have a high surface area/volume ratio, chemical reactivity is very high". In other words, nanomaterials are preferable for the manufacture of photocatalysts, sensors, solar cells, among others, since a greater specific surface area is directly related to greater efficiency of these devices. Physical and chemical characteristics of nanoparticles that include size, charge, shape and individual surface properties are important in the transport and distribution of active substances to interact with biomolecules, such as enzymes, antibodies and receptors, both on the surface and inside the cell.

In this context, Gracia-Pinilla et al. (2008) emphasized the importance of developing new nanomaterials for applications in medicine, electronics, optics and plasmonics and studied geometry models of regular polyhedra for silver nanoparticles that have antibacterial and antiviral properties. The authors used results from molecular dynamics simulations to compare the structural and energetic behavior of six different tetrahedron, icosahedron and cuboctahedron configurations. The objective was to relate the shape of the crystalline structure of silver nanoparticles to the thermodynamic conditions necessary for the stability of these structures.

Mckenna (2009) observed that the general trend is that with increasing size, in vacuum, the molecular organization of gold nanoparticles follows the following order of preference: decahedral, icosahedral, marked decahedral, and truncated octahedral. The author also highlighted that at high pressure, CO molecules are adsorbed on almost all surface atoms. This configuration returns nanoparticle morphologies with a larger surface area to be preferentially stabilized.

Bhatt and Kumar (2017) argue that it is very useful to understand the melting behavior of nanomaterials during their high-temperature applications. It is known that the melting temperature of metal, organic and semiconductor nanoparticles is lower compared to the same materials on a macroscopic scale. But, according to the authors, many models try to explain that the melting temperature is size-dependent, using the fact that nanoparticles are always ideal spheres.

However, as the size and shape of nanoparticles directly influence their volume and surface area, the difference between the specific surface area for nanoparticles of different shapes is very large. Thus, the authors generalized the applications of the binding energy model to study the effect of size on the melting temperature of independent nanosolids (silver - Ag; gold - Au; aluminum - Al; and zinc - Zn). These materials were chosen to enable comparison with experimental data available in other publications. The results demonstrated that the melting temperature of nanosolids decreases continuously, from the cylindrical to the tetrahedral shape, passing through the wire, spherical, and octahedral shape, in that order. The authors also found that the shape effect for overheating of lead (Pb) and Ag nanoparticles indicates that the melting temperature increases with decreasing particle size, and is highest for the tetrahedral shape and minimum for the cylindrical shape, and decreases in the following decreasing order: tetrahedral, octahedral, spherical, wire, icosahedral, and cylindrical.

The same authors also observed that when decreasing the size of nanoparticles, the total number of atoms and surface atoms decreases, but the specific surface area increases. Since the specific surface area also depends on the shape, the authors observed that it is minimum for the icosahedral shape and maximum for the tetrahedral shape. These data are in agreement with the specific surface area results of regular polyhedra (as discussed in the previous section). Thus, it is essential to consider the shape effect for developing these types of theoretical models.

Gisbert-Gonzalez et al. (2018) observed that the catalytic performance of metal nanoparticles regarding activity and selectivity in relation to the target reaction depends on the specificities of their surface, nature of their shape, and size and composition of these nanoparticles.

The authors report the need to refine methods that allow the synthesis of nanoparticles by observing these characteristics (composition, size and shape) to develop a catalyst and assist in solving challenges in sustainable chemistry (degradation of dyes, drug by-products, and heavy metals, under the influence of sunlight) and energy development (supercapacitors and photoelectrochemical cells). Experimental and computational methods have also been combined to obtain reported electrochemical results under different citrate concentrations and pH conditions (Gisbert-Gonzalez et al., 2018). The objective was to verify that, in neutral media, the citrate used as a covering agent in water originates platinum nanoparticles (electrocatalyst) of tetrahedral and octahedral shape in the water. The authors clarified the geometric characteristics of nanoparticles, bearing in mind that adsorption properties are, in general, quite dependent on the local surface specification, and that the newly deposited atom on the nanoparticle surface moves until it reaches a position where the energy is minimized.

Mineralogy and concrete technology

Exemplifying studies in Mineralogy, it is possible to visually recognize a mineral by observing its physical properties, such as color, shape, brightness, density and hardness. It is possible to highlight that pyrite stone have a shape similar to cube, octahedron or pentagonodecahedron. Galena (a primary lead ore that is used in the manufacture of lead-acid batteries) has the most common form of presentation in the truncated cube. Fluorite (used in the chemical, steel and metallurgical industries) has well-defined cube- or octahedron-shaped crystals. These volumetric systems are directly related to their respective crystalline forms at the level of atomic organization (Manzanares, 2023).

One of the objectives of Chaves' thesis (1997) was the classification of the main physical characteristics of diamonds. The author highlighted that, in general terms, in most cases the appearance of diamond crystals is octahedral or dodecahedral, sometimes cubic and, in very rare situations, resembling a tetrahedral shape. Crystals with curved faces are also common. Pereira (2010) developed experimental work to characterize iron oxides and fluid inclusions in rocky stone structures. According to the author, the basic structure of an iron oxide is an octahedron, in which each iron atom (2+ or 3+) is surrounded by six O^{2-} or six OH^{-}. The polymerization of these octahedra into compact arrangements, in which these units can interact through their apices, edges or faces, forms the crystalline structure of all minerals in this group. From the study of chemical structures, it is known that iron oxides are formed with anionic arrangements with interstices occupied with divalent or trivalent Fe, with the predominance of octahedral coordination FeO_6 (tetrahedral coordination FeO_4 may also be present). Iron oxides consist of anionic arrangements in which the interstices are partially filled with divalent or trivalent Fe, predominating octahedral FeO_6 coordination, however, tetrahedral FeO_4 coordination is also present. Each FeO_6 unit cell shares its edges with three other octahedra in the same plane, and one face is common with an octahedron in the adjacent plane.

It is known that mineral fragments contained in sedimentary deposits form soil particles. The shape of particles is an inherent characteristic of the soil and plays an important role in its physical properties and mechanical behavior. Regarding the size and shape of the particles, Ghasemi et al. (2019) studied the relationship between the specific surface area of the constituents in mortars and their consistency, and explored a hypothesis about the dependence of the workability of mortars on the packing density and the surface area of its constituents.

The authors clarified that the workability of mortars depends on the granulometric distribution and shape of the constituent particles, which interferes with the packing density and water demand of the mortars since denser packing leads to fewer voids between the particles that can be filled. Furthermore, they also highlighted that water demand is related to the surface area of the ingredients in the mixture, since mortars with aggregates that have a greater specific surface area and angular shape require more water for the same amount of cement.

Due to the complexity of characterizing the particle size distribution, it is assumed for mathematical simplification that the particles have a spherical shape. However, studies cited by the authors demonstrated errors of the order of 20-30% in estimating the specific surface area of the crushed material, justifying the proposition of a new methodology that uses dodecahedrons and hexahedrons (cubes) as uniform model shapes instead of spheres to improve the estimation of specific surface area and distinguish between natural and crushed aggregates. Similarly, Ghasemi et al. (2020) compared the specific surface area values calculated for different regular polyhedra whose edge length was defined in relation to the radius of intermediate spheres based on the particle size distribution obtained in the dry sieving method. For the authors, the importance of a better estimate of the specific surface area is due to the possibility of defining the shape quantitatively and providing information about the gradation and fineness modulus of the sampled aggregate. In this way, the possibility of replacing the assumption of a spherical shape with the shape of regular polyhedra was explored to improve the accuracy of estimating specific surface area of particles in the concrete mix, and classifying aggregates based on their angularity, representative shape and water demand for dosing mortars and concrete.

Biology

Haeckel (2012), seeking to relate the structure and shape of regular polyhedra with the similar shape of living organisms, highlighted the geometry of diatoms and radiolarian (small protozoa measuring 0.1-0.2 millimeters), found as zooplankton in solitary form and in colonies. Radiolarians are protozoa that are part of the marine plankton present in all oceans and that have external siliceous skeletons with polyhedral shapes. Coccoliths, tiny calcium carbonate platelets secreted by certain protozoa or algae in plankton that form colonies (coccolithophores), have a variety of shapes. Among them, it is possible to find those from the Braarudosphaeraceae and Goniolithaceae families, with a dodecahedron shape (Van Beveren, 2022; Samantha et al., 2020).

Also common are applications of mathematical structures, both algebraic and geometric, as well as systems organized at other scales, in the construction of codes of nature. The authors considered complexity and three-dimensionality to represent biological concepts such as translation, transcription, replication of the genetic code (DNA/RNA). The structure of the amino acids that make up DNA proteins, according to their chemical properties, has the shape of regular polyhedra (tetrahedron, hexahedron, icosahedron, dodecahedron and icosahedron). Lu, Guo, and Liu (2024) stated that one can find a single-stranded DNA molecule folded into a hollow octahedron, as well as other DNA polyhedra such as the tetrahedron, the cube and the truncated octahedron. In addition, icosahedral structures have been widely reproduced for virus capsids. According to Rivero (2019), the capsid is a structure that protects the genome of viruses and can have different shapes, such as icosahedral, like the adenovirus, and helical, like the tobacco mosaic virus.

Parvez (2020) reported that icosahedral and helical enveloped viruses are very common in animals and rare in plants and bacteria. This author also presented the structural and morphological diversity of viruses with notable differences in their shapes, sizes, molecular compositions and organizations. Rivero (2019) highlighted that the icosahedral structure is the largest structure that can accommodate 60 asymmetric subunits made of proteins and, furthermore, maximizes the amount of volume per unit area. However, studies cited by the author showed that the same symmetry was compatible with capsids composed of different numbers of subunits.

These fundamental protein units that make up the capsid, called capsomers, can be made up of one or more proteins of the same type, or even proteins of different types. In this sense, Lee et al. (2022) reported that capsomers cluster in groups of five (pentamers) around twelve equidistant revelations corresponding to the vertices of an icosahedron; and in groups of six (hexamers) around points at the intersection of the capsid with global, 3-fold and/or 6-fold local axes of symmetry of the same icosahedron. It has been highlighted that there are counted ways to form an icosahedral capsid. By replacing 12 hexamers with 12 pentamers, a closed surface is formed and the number of shapes that can be used to create an icosahedral capsid is related to the triangulation number, T. The number of proteins in a capsid or the total number of capsomeres in a capsid depends on the T value. Each of the structures with a different triangulation number can be found in various viruses, such as picornavirus, which has triangulation number T=3, or herpevirus, with triangulation number T=16.

Briefly, studying the atomic-level structure of various viruses with detailed 3D imaging obtained through high-resolution X-ray crystallography, cryo-electron microscopy and molecular simulations elucidates their morphological characteristics and enables the development of antiviral drugs (Mishra, 2020; Parvez, 2020). In resume, the knowledge of the viral capsid properties allows mechanical and chemical interference in their assembly or, more generally, in their replication cycle. Thus, Mishra (2020) clarifies that a mathematical study of the structure of viruses is necessary, as symmetry is used as a tool in virology to understand and classify viruses, considering that antiviral drugs are designed in such a way as to interfere or block viral assembly by interfering with the symmetry structure of the virus to render it inactive.

Robotics

The shape of regular polyhedra has been applied in new motion and rolling mechanism technologies for robots. According to Motahari-Bidgoli et al. (2014), the Tetwalker robot was designed at NASA Goddard Flight Center (GSFC) for space exploration, being able to maneuver smoothly on rocky and difficult terrain with stability, with just three supports. The Tetwalker is a tetrahedron-based mobile robot with telescopic struts and nodes. It falls across the landscape, deforming, as the top of the tetrahedron moves in the desired direction, while the rear edge moves in the same direction. As a result, its center of gravity changes and the robot falls. The process repeats as the robot continues to rotate towards its final location.

Most recently, using tetrahedron motion technology, Liu, Yao and Li (2020) developed a robot capable of converting rotational movements into exact rectilinear movements, which can be widely used in technological applications.

Based on tetrahedron-based walking robots, Ding and Yao (2013) propose a method for constructing an implantable hexahedral mobile mechanism. This mechanism can be considered an implant structure that can expand in six directions independently due to its completely symmetrical configuration. Stability analysis and dynamic simulation of walking and rolling were carried out and a prototype was manufactured to verify and validate experimental results of walking and tipping functions.

Meteorology and cartography

Modeling global atmospheric phenomena is of great scientific, economic, social and political importance. In Meteorology, as well as in several areas of knowledge, the sphere is used as a basis in weather forecasting and ocean modeling studies, for example. However, regular polyhedra can also serve as a basis for the development of nearly uniform meshes, since they can be inscribed in a sphere and their faces can be projected onto the surface so that the projections of their edges onto the sphere result in geodesic edges on the sphere. This type of mesh is called geodesic mesh for the sphere (Stasi, 2022).

The geodesic meshes obtained by projecting the edges of regular polyhedra onto the surface of the circumscribed sphere are uniform and do not present a concentration of points around the poles like the latitude-longitude meshes. In view of this, Silva et al. (2009) highlighted that several atmospheric models are being developed using these meshes. Stasi (2022) also highlights that the icosahedral type mech has gained a lot of attention among researchers due to its simpler geometric modeling patterns. In this geodesic mesh, the vertices of the regular icosahedron inscribed in a sphere are points of the geodesic mesh and are connected by geodesic arcs forming a triangular mesh composed of 20 geodesic triangles corresponding to the 20 faces of the regular icosahedron projected on the spherical surface.

A model called Ocean Land Atmosphere Model - OLAM) represented a new generation of meteorological models capable of simultaneously representing meteorological phenomena on a global scale (Silva et al., 2009). By coupling refined grids, the OLAM model can more accurately represent local-scale phenomena and estimate regional climate. This model uses an unstructured grid, in which the cells have a triangular shape in the horizontal direction, like an icosahedral geodesic mesh

Silva et al. (2009) also explains that in this model, increasing the spatial resolution of some region of interest can be obtained by dividing the triangular grids while maintaining, in the same location, the faces of the original triangle of the lower resolution grid. The same authors also highlight that increasing spatial resolution unlimitedly through mesh refinement (which can be done simultaneously in different regions) facilitates the understanding of meteorological and climate processes in these regions.

The geometric characteristics of regular polyhedra also enable their use in cartography. In this sense, due to the difficulty of representing the Earth's globe on a flat surface, reducing distortions in area, shape and angles, there are several types of cartographic projections. One of the projection systems divides the sphere into 20 equilateral spherical triangles, which are then flattened to form an icosahedron. This methodology is known as Buckminster Fuller projection or Dymaxion projection. This method comprises a projection carried out through the representation of the world on the surface of a polyhedron and results in a map with less visible distortion of the size and area of the continents. However, despite the few distortions in its representation of the Earth's surface, it is not possible to observe the distances between continents, and this makes its use in navigation systems impossible.

Final Considerations

The main concepts and geometric elements of regular polyhedra were presented and discussed, related to the modeling of phenomena in areas of basic and applied Science. The work showed that the research topic is current and has potential for the development of future research and interventions that consider the use of mathematical concepts to explain problem situations in other areas of knowledge.

It is expected that the results of this research can guide high school Mathematics and Science teachers in the development and execution of interdisciplinary pedagogical activities. Possible actions involve: the use of low-cost materials in the classroom to build polyhedra that help in the study of molecular structures and chemical bonds; proposal of workshops/short courses with manipulative materials to understand the importance of geometric concepts of polyhedra in Geography to facilitate the planning of the Earth's globe (such as the Buckminster Fuller projection, which reduces distortions in the area, shape and original angles of the world map); creation and execution of viable experimental routes for the synthesis of nanoparticles (such as metal nanoparticles and semiconductor metal oxides) with specific polyhedral shapes, and testing their antimicrobial and/or photocatalytic actions in the degradation of contaminants present in water (for example, remaining dyes from industrial activities); knowledge of characterization techniques (X-ray diffraction, Raman spectroscopy, etc.) and measurement equipment (in non-formal teaching environments, such as in research institute laboratories), which are used to identify microorganisms and crystalline and molecular structures of nanomaterials, and to understand their properties depending on the polyhedral shape; development and execution of robotic projects that address the geometric characteristics of polyhedra to solve practical problems involving the requirement for locomotion and rolling.

In short, these actions aim to explore an interdisciplinary approach to Mathematics and Science and a defragmented teaching of reality, in a more attractive and contextualized way, with a view to more meaningful scientific and technological learning in Basic Education.

Estruturas de Poliedros Regulares: conceitos matemáticos e aplicações nas Ciências

Resumo

O ensino explanado de forma interdisciplinar representa uma importante ferramenta para a aprendizagem significativa em Matemática, com vistas à contextualização e aplicação de conceitos em outras áreas do conhecimento. Neste contexto, as regulamentações educacionais oficiais no Brasil na área da Matemática e suas Tecnologias defendem a construção de uma visão mais integrada da Matemática e sua relação com a Ciência. Dentre as competências destacadas para o ensino de Matemática na Educação Básica, destacam-se a interpretação de fenômenos e a solução de situações-problema a partir de conceitos de poliedros regulares. Embora seja conhecida a importância deste tema para a melhoria do ensino e aprendizagem de Matemática, as aplicações dos poliedros regulares ainda são pouco exploradas no ambiente escolar e até mesmo na literatura. Nesse sentido, este trabalho apresenta uma revisão de literatura, de natureza exploratório-descritiva, com o objetivo de destacar os poliedros regulares na modelagem de fenômenos em áreas científicas e tecnológicas para utilização dessa relação no ambiente educacional. Os resultados mostraram que os elementos geométricos e as propriedades dos poliedros *regulares são utilizados para modelar e entender fenômenos em áreas da ciência básica (biologia, química e materiais) e aplicada (como em robótica, cartografia, meteorologia e mineralogia), significando uma abordagem interdisciplinar para a melhoria do Ensino de Matemática e de Tecnologias no Ensino Médio.Inserir aqui o abstract do artigo. Sua redação deve ser paralela à do resumo.*

Palavras-chave

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